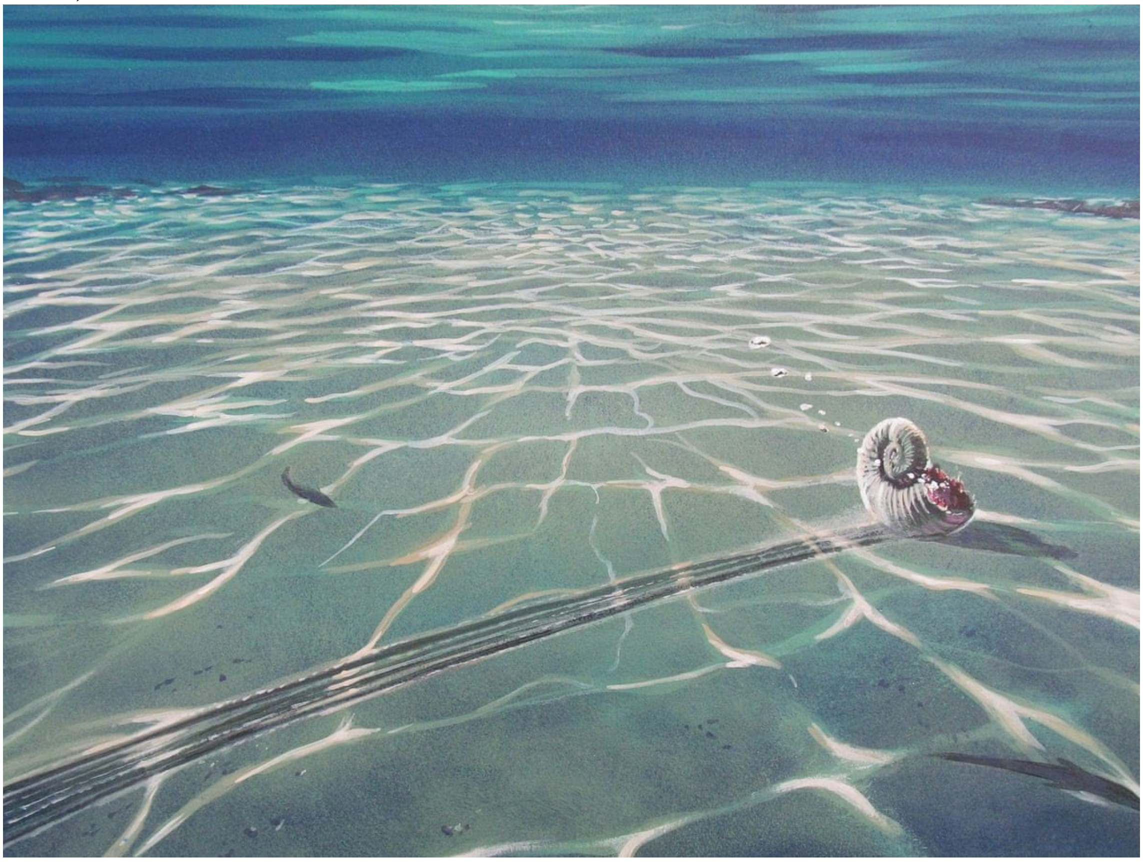
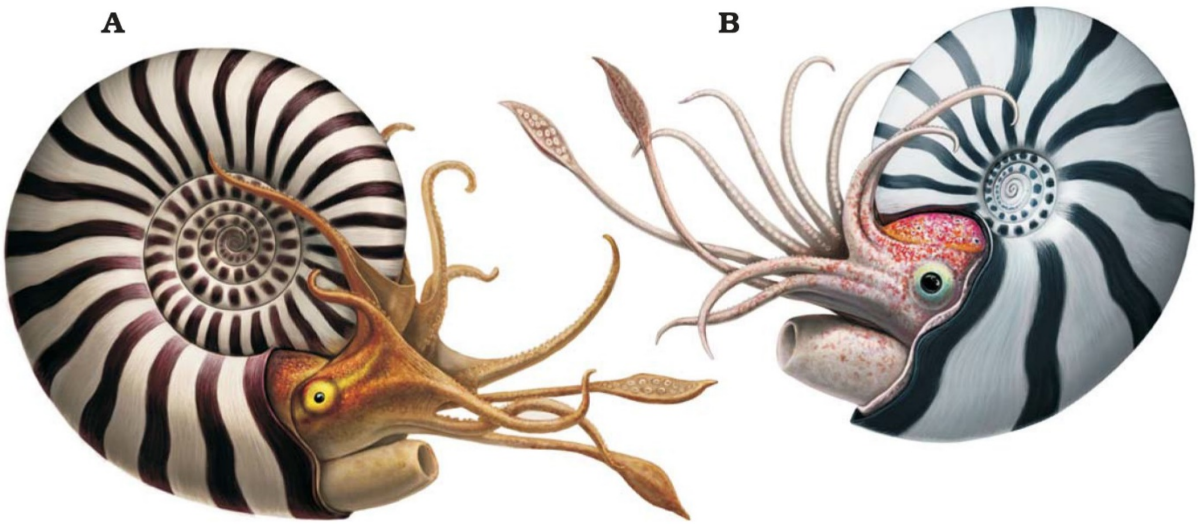
Claws, shells and Fibonacci Spirals

As part of our special "Dino-maths Day", I want us to focus on another important group of prehistoric organisms that also perished, around 65 million years ago, during Earth's most recent mass extinction event…ammonites!

When they were alive, they would have enjoyed a relatively predator-free existence in warm, shallow tropical seas; we only find the hard (fossilised) shells, these days, but the shells would have offered substantial protection for the soft cephalopod (a class of mollusc) that lived inside.



Here are some illustrations of the animals that would have lived inside the string calcium carbonate shells.

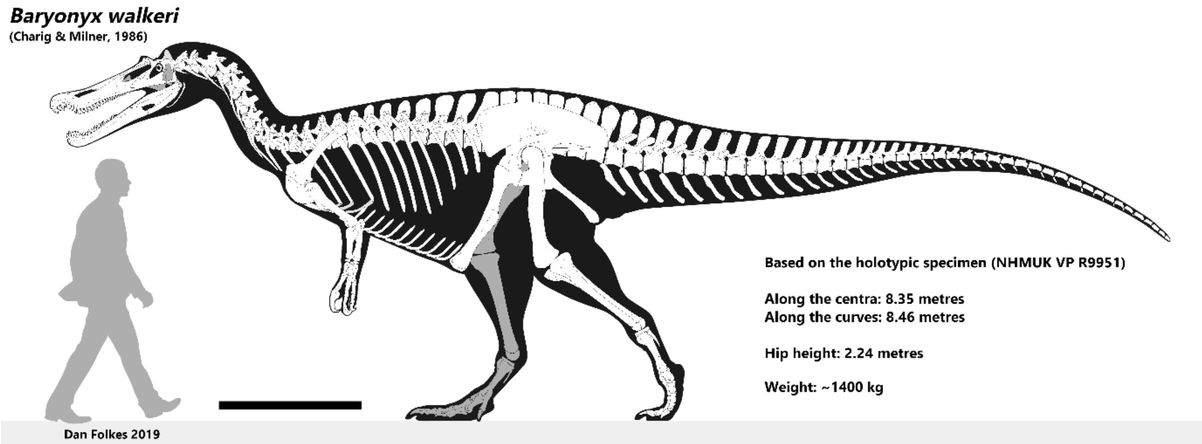


What do you notice about the shape of the shells? [Hint: cast your eyes around from the centre of the shell and anticlockwise until you get to the large aperture where the ammonite emerges to feed and propel itself along underwater]

It may help if you compare the above spiral with these examples below.

A snake's body is, generally, the same thickness along its length so, just like the coiled rope, the thickness of the spiral that both will form will be constant; this is known as an Archimedes spiral.
However, a cephalopod will, like all animals, grow as the animal gets older; so, the shell must continually increase in size to accommodate its occupant; this is referred to as a Fibonacci spiral.

Sadly, this magnificent reptile (Baryonyx walkerii) no longer exists.



Aside from being a ferocious predator, it also had a reputation for not clipping its toe nails frequently. It had claws that also grew in the form of a Fibonacci spiral.

We can produce a somewhat angular Fibonacci spiral quite easily by taking a square of card and successively dividing it into ever-decreasing isosceles, right-angled triangles. It can, with care, turn into really eye-catching display work. [Examples will be available to view].

Claws, Shells and Fibonacci Spirals.

Numerator	Denominator	Numerator/Denominator
1	1	1.0000
2	1	2.0000
3	2	
5	3	
8	5	
13	8	
21	13	
34	21	
55	34	

Claws, Shells and Fibonacci Spirals.

Numerator	Denominator	Numerator/Denominator
89	55	
144	89	
233	144	
377	233	
610	377	
987	610	
1597	987	
2584	1597	
4181	2584	

Claws, Shells
and Fibonacci Spirals.

LO: to develop an understanding of non-linear sequences and how terms within these sequences are related.

Any group of numbers (two or more) can be set out in numerical order to form a *sequence*.
***Ascending numerical order* means smallest to largest.**

For example:
3, 6, 9, 12, 15,…
Are the first five *terms* in what well-known number sequence?
3, 6, 9, 12, 15,…
Are the first five *terms* in the "three times table".
What about 7, 14, 21, 28, 35,…?

7, 14, 21, 28, 35,…
Are the first five terms in the sequence that could be called "multiples of seven".
The difference between each pair of consecutive terms will always be 7.
What about…
9, 16, 23, 30, 37,…?

Although the terms in each of these sequences is different, both these sequences have something in common.
What is it?
To get the next term, you simply add 7.

If you only have to add (or subtract) the same value each time, you will generate a *linear sequence*.
Let's look at some more examples…

a) **4, 9, 14, 19, 24,_,_,_,_,…**
b) **5, 16, 27,_,_,_,_,…**
c) **101, 107, 113, 119,_,_,_,_,…**
d) **-12, -7, -2, 3,_,_,_,_,…**
e) **100, 92, 84, 76,_,_,_,_,…**

Handy Hint #57
Always look for patterns when you generate a number sequence;
It can often help you with your calculations.

For this sequence, the "rule" is double each term to get the next one (or multiply by two).
2, 4, 8, 16,_,_,_,_,…
For this sequence, the "rule" is halve each term to get the next one (or divide by two).
240, 120, 60,_,_,_,_,…

You have already encountered some sequences that are non-linear (the difference between each pair of terms changes).
1, 4, 9, 16, 25, 36,…
Are the first six terms in the sequence of square numbers.

1, 3, 6, 10, 15, 21,…
Are the first six terms in the sequence of triangular numbers.

1, 8, 27, 64, 125, 216,…
Are the first six terms in the sequence of cube numbers.

Let us now dwell on another well-known non-linear sequence…
0, 1, 1, 2, 3, 5, 8, 13,_,_,_,_,_,_,_,…
This is known as a Fibonacci Sequence.
How do you get the next term in the sequence?
To get the next term in the sequence, you must add the previous two terms.
You will have noticed that the numbers increase dramatically as the sequence continues….